Week 5 PYQs
Machine Learning Foundations
May 22 - Sep 25

Note

1. If no questions are listed in that term Quiz, it indicates that the corresponding quiz did not include questions from that week.

2. The answer key may contain minor inaccuracies, as answers were taken directly from the original quiz/exam papers. Some of these might have been corrected later.

3. For any issues or suggestions regarding this document, feel free to contact:
   `23f3004032@ds.study.iitm.ac.in`
   or reach out on Discourse: `@23f3004032`

Quiz 1 May 22

Quiz 2 May 22

1. Which of the following is/are eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 1+i \\ -1+i & 6 \end{bmatrix}?$$

(a) $\begin{bmatrix} \frac{1}{2} \\ \frac{-1+i}{2} \end{bmatrix}, \begin{bmatrix} 1 \\ 1-i \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} \\ \frac{-1+2i}{2} \end{bmatrix}, \begin{bmatrix} 1 \\ 1-i \end{bmatrix}$

(c) $\begin{bmatrix} 1+i \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1-i \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{2} \\ \frac{-1-2i}{2} \end{bmatrix}, \begin{bmatrix} 1 \\ 1-i \end{bmatrix}$

[Ans: A, C]

2. Let A be an $n \times n$ Hermitian matrix. Suppose $A = UDU^*$, all diagonal entries of D are 1. Then which of the following statements is/are true?

   (a) A is the identity matrix.

   (b) A can be a matrix other than identity matrix.

   (c) Any vector in $v \in \mathbb{C}^n$ is an eigenvector of A with eigenvalue 1.

   (d) Not all vectors in $v \in \mathbb{C}^n$ will be eigen vectors of A.

   (e) $A = D$

[Ans: a,c,e]

ET May 22

1. If two complex vectors v_1 and v_2 have lengths l_1 and l_2 and $v_1 \cdot v_2 = l_1 l_2$, then which of the following statements is/are true?

   (a) v_1 and v_2 vectors are parallel to each other.

   (b) v_1 and v_2 vectors are conjugate to each other.

   (c) v_1 and v_2 vectors are perpendicular to each other.

   (d) v_1 and v_2 can be any two vectors.

   [Ans: 1]

2. If two complex vectors v_1 and v_2 have lengths l_1 and l_2 and $v_1 \cdot v_2 = l_1 l_2$, then which of the following statements is/are true?

   (a) v_1 and v_2 vectors are parallel to each other.

   (b) v_1 and v_2 vectors are conjugate to each other.

   (c) v_1 and v_2 vectors are perpendicular to each other.

   (d) v_1 and v_2 can be any two vectors.

   [Ans: 1]

Quiz 1 Sep 22

Quiz 2 Sep 22

1. Choose the correct option(s) with respect to a square Hermitian matrix A of order n.

   (a) If A is a Hermitian matrix, and k is any real scalar, then kA is also a Hermitian matrix.

   (b) For every $a_{ij} \in A$, $\overline{a_{ij}} = a_{ji}$ for all $(1 \leq i, j \leq n)$.

   (c) Every diagonal element of the matrix A is a real number.

   (d) The determinant of A can either be real or complex.

   [Ans:a,b,c]

2. Consider a Hermitian matrix

$$A = \begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix}$$

   and choose the correct option(s):

   (a) The eigenvalues of A are 1 and 3

   (b) The eigenvalues of A are 1 repeated twice

   (c) The matrix A is unitarily diagonalizable

   (d) The matrix A is not unitarily diagonalizable

   [Ans: A, C]

3. The matrix

$$A = \frac{1}{2}\begin{bmatrix} 1+i & \sqrt{k} \\ 1-i & \sqrt{k}i \end{bmatrix}$$

   is unitary if k is

   (a) $\frac{1}{2}$

   (b) 1

   (c) 2

   (d) $\frac{1}{4}$

   [Ans:2]

ET Sep 22

1. Which of the following statements is/are false?

   (a) Rank of a $n \times n$ Hermitian matrix is always equal to n.

   (b) Rank of $n \times n$ unitary matrix is always equal to n.

   (c) If A is a Hermitian matrix, then A^T is Hermitian.

   (d) Zero can be one of the eigenvalues of a unitary matrix.

[Ans: 1, 4]

Quiz 1 Jan 23

Quiz 2 Jan 23

1. Given three unitary matrices A, B, C, which are true?

   (a) ABC is always unitary

   (b) $A + B$ is Hermitian

   (c) AB, BC, AC are unitary

   (d) ABC may not be unitary

   [Ans: A, C]

2. Eigenvalues of a matrix that is both unitary and Hermitian:

   (a) 0

   (b) 1

   (c) -1

   (d) i

   (e) 2

   (f) -2

   [Ans: 1, -1]

3. Let

$$A = \begin{bmatrix} 1 & -i & -1 \\ i & -1 & -i \\ -1 & i & -1 \end{bmatrix}$$

   What is the unitary diagonalization of A?

   (a)
$$\begin{bmatrix} -2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -i/\sqrt{6} & -i/\sqrt{3} & i/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} -2/\sqrt{6} & i/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & i/\sqrt{3} & 1/\sqrt{3} \\ 0 & -i/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

   (b)
$$\begin{bmatrix} -2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & -i/\sqrt{3} & i/\sqrt{2} \\ i/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} -2/\sqrt{6} & -1/\sqrt{6} & i/\sqrt{6} \\ 1/\sqrt{3} & -i/\sqrt{3} & 1/\sqrt{3} \\ 0 & i/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

   (c)
$$\begin{bmatrix} -2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & -i/\sqrt{3} & i/\sqrt{2} \\ i/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -2/\sqrt{6} & -1/\sqrt{6} & i/\sqrt{6} \\ 1/\sqrt{3} & -i/\sqrt{3} & 1/\sqrt{3} \\ 0 & i/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

(d)

$$\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 0 \\ 0 & -i/\sqrt{3} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 0 & i/\sqrt{2} \\ 1/\sqrt{3} & -i/\sqrt{3} & 1/\sqrt{3} \\ 0 & i/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

[Ans:a]

4. Which of the following matrices are both Hermitian and unitary?

(a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[Ans:a,b,d]

ET Jan 23

1. Which of the following statements is/are true?

   (a) If A is a hermitian matrix, then eigenvectors of matrix A are also eigenvectors of matrix A^n, where $n = 2, 3, 4, \cdots$

   (b) If λ is an eigenvalue of matrix A, then λ will also be the eigenvalue of matrix A^n.

   (c) If λ is an eigenvalue of matrix A, then λ^n will be the eigenvalue of matrix A^n.

   (d) If A is a hermitian matrix and x is an eigenvector of matrix A then nx is an eigenvector of matrix A^n, where $n = 2, 3, 4, \cdots$

   [Ans: 1, 3, 4]

Quiz 1 May 23

Quiz 2 May 23

1. Which among the following is/are true for a Hermitian matrix?

   (a) The eigenvalues of a Hermitian matrix are always real.

   (b) The diagonal elements of a Hermitian matrix are always real.

   (c) All symmetric matrices are Hermitia

   (d) All Hermitian matrices are symmetric

   [Ans: A, B]

2. Which among the following statements is/are true?

   (a) If a function is positive semidefinite, then it only has a global minimum.

   (b) If a function is positive semidefinite, then it has both global minimum and global maximum.

   (c) If a function is negative semidefinite, then it only has a global maximum.

   (d) If a function is negative semidefinite, then it has both global minimum and global maximum.

   [Ans: a, c]

3. Consider a 2×2 matrix

$$A = \frac{1}{k}\begin{bmatrix} 2 & -2+i \\ i+2 & 2 \end{bmatrix}$$

   Find the value of k such that A is a unitary matrix.

   [Ans: 3]

ET May 23

1. Let A and B be two $n \times n$ complex Hermitian matrices. Then, which among the following are true?

   (a) $A + B$ is Hermitian.

   (b) AB is Hermitian.

   (c) $iA + (2 + i)B$ is Hermitian.

   (d) $A + iB$ is Hermitian.

[Ans: 1]

Quiz 1 Sep 23

Quiz 2 Sep 23

1. Let U andV be two unitary matrices. Consider the following statements:
   Statement 1: U + V is unitary. Statement 2: UV is unitary.
   Select the correct options from the following:

   (a) Only statement 2 is correct.
   (b) Both statement 1 and statement 2 are incorrect.
   (c) Both statement 1 and statement 2 are correct.
   (d) Only statement 1 is correct.

   [Ans:a]

2. Let
$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \in \mathbb{C}^{2\times 2}$$
   Which of the following statements are true?

   (a) A is Hermitian matrix
   (b) A is not Hermitian matrix
   (c) A is unitarily diagonalizable
   (d) A is not unitarily diagonalizable

   [Ans: 2, 3]

ET Sep 23

Quiz 1 Jan 24

Quiz 2 Jan 24

1. Consider two complex matrices A and B such that both are Hermitian matrices. Which of the following statements is/are true?

   (a) $A + B$ is also a Hermitian matrix

   (b) $\overline{A+B}$ is a Hermitian matrix, where $\overline{A+B}$ denotes the conjugate of $A + B$

   (c) AB is also a Hermitian matrix

   (d) $\overline{AB}$ is a Hermitian matrix, where $\overline{AB}$ denotes the conjugate of AB

   [Ans: 1, 2]

2. Consider a matrix

$$A = \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$$

   Use this information to answer the given sub-questions.

   (a) Which of the following statements is/are true about A?

      i. A is the Hermitian matrix.
      ii. A is the Unitary matrix.
      iii. All eigenvalues are real.
      iv. Eigenvalues are complex.

   [Ans: A, C]

   (b) Which of the following statements is/are true about A?

      i.
$$A = \begin{bmatrix} \frac{-i\sqrt{2}}{2} & \frac{i\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{-i\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{i\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

      ii.
$$A^2 = \begin{bmatrix} \frac{-i\sqrt{2}}{2} & \frac{i\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{-i\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{i\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

      iii.
$$\begin{bmatrix} i \\ 1 \end{bmatrix} \text{ is an eigenvector of } A$$

      iv. A is not diagnolizable

   [Ans: A, B, C]

3. Which of the following statement is true about the matrix

$$A = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}$$

(a) A is positive definite

(b) A is positive semi-definite

(c) A is negative definite

(d) A is negative semi-definite

[Ans: 1]

ET Jan 24

Quiz 1 May 24

Quiz 2 May 24

1. Consider matrix

$$A = \begin{bmatrix} 1 & 1+i & 2i & 9 \\ 1-i & 3 & 4 & 7-i \\ -2i & 4 & 5 & i \\ 9 & 7+i & -i & 7 \end{bmatrix}$$

Which of the following options is/are true?

(a) A is a Hermitian matrix

(b) A is diagonalizable

(c) All eigenvalues of A are real

(d) A has eigenvalues that are neither real nor purely imaginary

[Ans: 1, 2, 3]

2. Consider

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{bmatrix}$$

Which of the following options is/are true?

(a) A is a unitary matrix

(b) $\|Av\| = \|v\|$ may or may not hold

(c) Eigenvectors of A are orthogonal

(d) If α is an eigenvalue, then $|\alpha| = 1$

[Ans: 1, 3, 4]

ET May 24

Quiz 1 Sep 24

Quiz 2 Sep 24

1. Consider
$$B = \begin{bmatrix} 4 & -2i & -1 \\ 2i & 0 & -i \\ -1 & i & 4 \end{bmatrix}$$
(Note: B^H denotes conjugate transpose)
Which of the following options is/are true?

   (a) $B^2 = -B^H$
   (b) $B = B^H$
   (c) Every eigenvalue of B is real
   (d) $\det(B) = 0$

[Ans: 2, 3]

2. Consider a unitary matrix A. Let α be an eigenvalue. Which of the following is/are true?

   (a) $|\alpha| = 1$
   (b) α is an eigenvalue of A^H
   (c) Eigenvectors of A are orthonormal
   (d) A is not diagonalizable

[Ans: 1, 2, 3]

3. Suppose
$$A = \begin{bmatrix} 1+i & 2 \\ 0 & 3+i \end{bmatrix}$$
Which of the following represent eigenvectors of A?

   (a) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 \\ -i \end{bmatrix}, \begin{bmatrix} i \\ i \end{bmatrix}$
   (c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}$
   (d) $\begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -i \\ -i \end{bmatrix}$

[Ans: 1]

ET Sep 24

1. Consider a matrix $A = \begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix}$. Which of the following options is/are true?

   (a) A is not a unitary matrix.

   (b) A is Hermitian matrix.

   (c) Eigenvectors of A are orthogonal.

   (d) Eigenvalues of A are not real.

[Ans: 3, 4]

Quiz 1 Jan 25

Quiz 2 Jan 25

1. Consider the Hermitian matrix $A = \begin{bmatrix} 3 & i \\ -i & 5 \end{bmatrix}$. Which of the following are the eigenvalues of A?

   (a) 3 and 5

   (b) $4 + \sqrt{2}$ and $4 - \sqrt{2}$

   (c) $4 + i$ and $4 - i$

   (d) 2 and 6

   [Ans: 2]

2. Consider the complex matrix: $A = \begin{bmatrix} 1 & 2+i \\ 0 & 3 \end{bmatrix}$. Using Schur's theorem, which of the following is the correct upper triangular form of A after a similarity transformation?

   (a) $\begin{bmatrix} 1 & 2+i \\ 0 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 \\ 2+i & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & x \\ 0 & 3 \end{bmatrix}$ for some complex x.

   [Ans: 1]

3. Let A be a complex square matrix. What is the relationship between the eigenvalues of A and its conjugate transpose A^*?

   (a) The eigenvalues of A^* are the complex conjugates of the eigenvalues of A.

   (b) The eigenvalues of A^* are the same as the eigenvalues of A.

   (c) The eigenvalues of A^* are the reciprocals of the eigenvalues of A.

   (d) The eigenvalues of A^* correspond to the square roots of the eigenvalues of A.

   [Ans: 1]

4. Consider a matrix $A = \begin{bmatrix} 1 & 1 & i \\ 1 & -1 & -i \\ -i & -i & 1 \end{bmatrix}$. Which of the following options is/are true?

   (a) A is a unitary matrix.

(b) $A = A^*$

(c) Every eigenvalue of A is real.

(d) The determinant of A is zero.

[Ans: 2, 3]

5. Which of the following statements is/are True regarding Schur's Theorem?

(a) Every square matrix is unitarily similar to an upper triangular matrix.

(b) The diagonal entries of the Schur form of a matrix are its eigenvalues.

(c) A square matrix with repeated eigenvalues cannot always be unitarily similar to an upper triangular matrix.

(d) Schur's Theorem holds only for square matrices, as it requires unitary similarity and the presence of eigenvalues.

[Ans: 1, 2, 4]

ET Jan 25

Quiz 1 May 25

this was term in which syallabus was week 1-5 for quiz 1

1. Consider a matrix
$$A = \begin{bmatrix} 1 & i & -i \\ i & 1 & 0 \\ -i & 0 & 1 \end{bmatrix}.$$
Based on this information, answer the given subquestions.

   (a) Which of the following options is/are true?
   - i. $A^* = A$, where A^* is the conjugate transpose of matrix A.
   - ii. A is a symmetric matrix.
   - iii. A is a unitary matrix.
   - iv. Columns of A are not orthogonal.

   [Ans: 2,4]

   (b) Which of the following options is/are true?
   - i. 1 is an eigenvalue of A.
   - ii. $1 + i\sqrt{2}$ is an eigenvalue of A.
   - iii. The rank of the matrix A is 2.
   - iv. Nullity of the matrix A is 1.

   [Ans:1 ,2]

   (c) Which of the following options is/are true?
   - i. Matrix A is diagonalizable.
   - ii. Eigenvectors of A are orthogonal.
   - iii. If A is diagonalizable and diagonal form of $A = P^{-1}DP$, then P is an orthogonal matrix.
   - iv. Entries of the eigenvector v corresponding to the eigenvalue $1 - i\sqrt{2}$ of A are complex numbers.

   [Ans:1,2,3]

Quiz 2 May 25

1. Consider a matrix $A = \begin{bmatrix} i & 1 \\ -1 & i \end{bmatrix}$. Which of the following options is/are true?

   (a) A is a Hermitian matrix.

   (b) A is a unitary matrix.

   (c) All eigenvalues of A are not real.

   (d) Eigenvectors corresponding to different eigenvalues of A are not orthogonal.

   [Ans: 2, 3]

2. Consider a Hermitian matrix A. Let α and β be the eigenvalues of A. Which of the following options is/are true?

   (a) α and β both are real.

   (b) If $\alpha \neq \beta$, then the corresponding eigenvectors are linearly independent.

   (c) If $\alpha \neq \beta$, then the corresponding eigenvectors are orthogonal.

   (d) A is not diagonalizable.

   [Ans: 1, 2, 3]

ET May 25

1. Let $A = \begin{bmatrix} i & 0 & 0 \\ -i & i & 0 \\ 1 & 0 & -i \end{bmatrix}$. Based on the above data, answer the given subquestions.

   Suppose λ_1, λ_2 and λ_3 are the eigenvalues of $2A^2 - 3I$,

   (a) find $\lambda_1\lambda_2\lambda_3$ [Ans: -125]

   (b) Which of the following options is /are true?

   i. A is not a Hermitian matrix.

   ii. A is not a unitary matrix.

   iii. A is diagonalizable.

   iv. None of these

   [Ans: 1, 2]

Quiz 1 Sep 25

Quiz 2 Sep 25

1. If A is Hermitian, then eigenvectors corresponding to distinct eigenvalues are orthogonal.

   (a) True

   (b) False

   [Ans: 1]

2. Consider a unitary matrix U as given below: $U = \frac{1}{2}\begin{bmatrix} 1+i & x \\ 1-i & y \end{bmatrix}$. Also, $\frac{x}{y} = a + ib$ and $a, b \in \mathbb{R}$. Enter the value of a. [Ans: -0.1 to 0.1]

3. Enter the value of b for the matrix U defined in the previous question. [Ans: -1.1 to -0.9]

4. Let S be the set of all unitary matrices U which have the form given in the question, that is, the first column of U is the vector $(\frac{1+i}{2}, \frac{1-i}{2})^T$. Which of the following is true?

   (a) S is an infinite set.

   (b) S is a finite set that has at least two elements.

   (c) S is an empty set.

   (d) S has exactly one element.

   [Ans: 1]

ET Sep 25